

IIR Filter

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1 Introduction

Yesterday you have got some experience operating the LLRF4 board, looking at the analog-to-digital converter (ADC) signals and generating some synthesized output waveforms. Today we will connect the ADC to the digital-to-analog converter (DAC) via a digital filter. The filter implemented in the LLRF4 board for this class is a second-order infinite impulse response (IIR) design (biquad). More information on the IIR filter and its implementation can be found in Section 5 of the LLRF4 lab reference manual.

2 IIR Coefficients

In this section you will find a short discussion of the relationship between the coefficients of the second-order polynomial and the roots. Let's consider the denominator polynomial of the second-order IIR filter as shown in Eq. 1

$$D(z) = z^2 + a_1z + a_2 \quad (1)$$

As you well know, this polynomial has two roots. They can be either both real or both complex. If the roots are complex, then $p_1 = p_2^*$, that is one root is the complex conjugate of the other. We will limit our discussion to the case of complex roots. In polar coordinates the roots can be expressed as:

$$p_{1,2} = re^{\pm i\theta} \quad (2)$$

For a stable IIR filter radius r has to be smaller than 1. Let's rewrite the polynomial $D(z)$ as

$$D(z) = (z - p_1)(z - p_2) = z^2 - p_1z - p_2z + p_1p_2 \quad (3)$$

Substituting root expressions from Equation 2 into Eq. 3 we get:

$$D(z) = z^2 - 2r \cos \theta z + r^2$$

This expression allows one to obtain coefficients a_1 and a_2 from the polar coordinates of the roots. Recall that the frequency response is evaluated at $z = e^{i\omega}$, where $\omega = 2\pi f/f_s$ is the normalized frequency. When $\omega = \theta$ the magnitude of the polynomial is minimal, so IIR denominator becomes small,

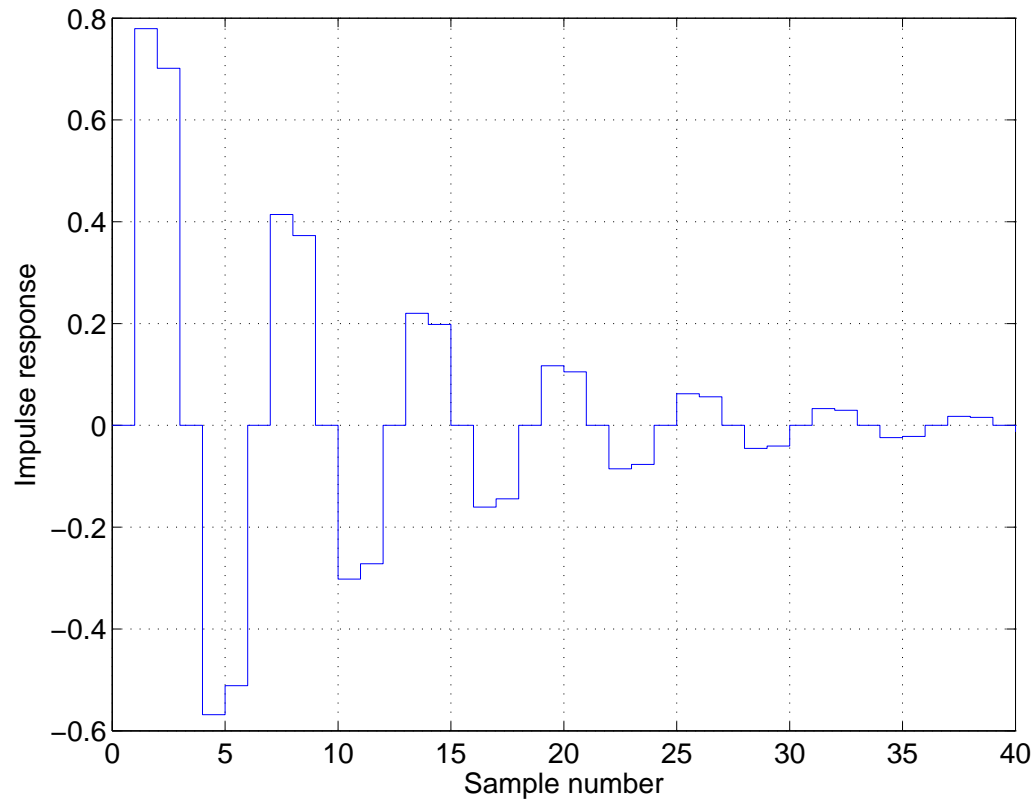


Figure 1: IIR filter impulse response.

signifying a gain peak. Pole radius defines the magnitude of the peak — for $r = 0.9$ we expect a gain of 10 (given by $1/|D(z)| \approx 1/0.1$). Also, the radius is directly related to the damping time of the filter response. Impulse response of an **IIR** filter with transfer function¹

$$H(z) = \frac{r \sin \theta z}{z^2 - 2r \cos \theta z + r^2} \quad (4)$$

is given by a sequence

$$h[n] = r^n \sin \theta n u[n] \quad (5)$$

¹To obtain such a response from LLRF4, set $b_1 = r \sin \theta$, $a_1 = -2r \cos \theta$, $a_2 = r^2$

where $u[n]$ is a unit step sequence, defined as 0 for $n < 0$, 1 otherwise. Such response for $r = 0.9A$ and $\theta = \pi/3$ is plotted in Figure 1

The damping time constant of such impulse response can be expressed as:

$$\tau = -\frac{T_s}{\ln r} \quad (6)$$

3 Exercises

3.1 Measuring Through Response

Our first measurement is very basic and will help you verify that everything is connected correctly and is operational.

Connect the network analyzer stimulus port to **ADC0**. Connect the response port to **DAC1**. Set stimulus power level (under **Sweep Setup**) to +9 dBm.

At this point you should be familiar with starting the experimental physics and industrial control system (**EPICS**) interface and making sure the board was operational. Turn on **ADC0** and turn off the other three channels. Turn on the **DACs**. Select **DAC1** signal as **IIR**. On the **IIR** coefficient panel, set b_0 and g_0 to 1, all other coefficients to 0. The frequency response of the filter, plotted in the window, should have 0 dB gain and zero phase shift — we are just passing through the signal.

Set up the network analyzer to sweep from 40 to 56 MHz — the response band of the **ADC** input filter. Your magnitude response should show the shape of that filter now. Record peak gain and measure the bandwidth of the filter:

3.2 Resonant Response

Now configure the filter coefficients for peak response at 48 MHz and the damping time of 75 ns². Note, that numerator coefficient b_1 could be considered as an arbitrary gain, so there is no need to compute $r \sin \theta$. The resulting filter has a gain of 10 at the peak, so it will probably saturate. Reduce gain G_0 to 0.1 or lower to avoid saturation. Record peak gain and measure the bandwidth of this digital filter³:

Now adjust coefficients to the damping time of 8 μ s. Remember to reduce the gain if saturation occurs. At this point you might have to reduce the IF bandwidth on the network analyzer to get the accurate sweep. Record peak gain and measure the bandwidth and quality factor of this digital filter:

²Hint: use Equations 4 and 6.

³In all of these measurements, feel free to re-center the network analyzer sweep on the peak and to zoom in for a closer look. Use 1601 points in a sweep for better peak definition.

3.3 Negative Group Delay

In this exercise we will configure the LLRF4 board for a response with negative group delay. Set the denominator coefficients a_1 and a_2 to 0. Set the gain to 1. Now compute the polynomial coefficients with the roots at the radius of 0.99 and the angle corresponding to 48 MHz. We need to configure the numerator (b_0 , b_1 , and b_3) to this polynomial. However these coefficients cannot exceed 1 and you probably have a value for b_1 larger than one. Scale all three coefficients by 0.5, so that $b_0 = 0.5$ and so on. Enter the values and observe the response. You should see a notch in $|S_{21}|$ at 48 MHz. Add a trace on the network analyzer displaying the group delay. Measure the minimum group delay observed⁴.

Change trace format to phase. Does the phase plot look similar to what you see on the [EPICS](#) panel?

3.4 Extra Credit

Try to adjust [IIR](#) filter parameters to generate as sharp a response as possible⁵. You will have to reduce the gain as you narrow the response down. At some point severe quantization effects in the filter structure will prevent you from going any further. Measure and record the center frequency, peak gain, bandwidth, and Q:

⁴Use averaging and set IF bandwidth to 7 kHz or less

⁵Hint: get r closer to 1

4 Glossary

Glossary

analog-to-digital converter (ADC)

An electronic circuit that converts continuous analog signals to discrete digital numbers. [2](#), [4](#)

digital-to-analog converter (DAC)

A hardware device to convert a sequence of digital codes to corresponding analog voltages or currents. [2](#), [4](#)

experimental physics and industrial control system (EPICS)

A set of software tools and applications used to develop distributed soft real-time control systems. [4](#), [6](#)

finite impulse response (FIR)

A discrete-time filter, output of which only depends on a finite number of previous input samples. [7](#)

infinite impulse response (IIR)

A discrete-time filter, output of which depends on an infinite number of previous input samples. Unlike finite impulse response ([FIR](#)) filters, IIR structures involve internal feedback. [2](#), [4](#), [6](#)